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THE STABILITY OF TWO-DIMENSIONAL WAKES AND SHEAR-LAYERS AT HIGH MACH NUMBERS[r]

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ABSTRACT

This study is concerned with the stability properties of laminar free shear-layer flows, and in particular symmetric two-dimensional wakes, for the subsonic through the hypersonic regime. Emphasis is given to the use of proper wake profiles that satisfy the equations of motion at high Reynolds numbers. In particular we study the inviscid stability of a developing two-dimensional wake as it accelerates at the trailing edge of a splitter plate. The non-parallelism of the flow is a leading order effect, and the undisturbed state is solved numerically. The neutral stability characteristics are computed numerically and the hypersonic stability is obtained by increasing the Mach number. It is found that the neutral stability characteristics are altered significantly as the wake develops. Multiple modes (second modes) are found in the near-wake (they are shown to be closely related to the corresponding Blasius ones), but as the wake develops mode multiplicity is delayed to higher and higher Mach numbers. At a distance of about one plate length from the trailing edge, there is only one mode in a Mach number range of zero to twenty. The dominant mode emerging at all wake stations and for high enough Mach numbers is the so-called vorticity mode, which is centered around the generalized inflection point layer. The structure of the dominant mode is also obtained analytically for all streamwise wake locations and it is shown how the far-wake limit is approached. Asymptotic results for the hypersonic mixing layer given by a tanh and a Lock distribution are also given.

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1. Introduction.

Recently there has been a renewed interest in hypersonic flows, mainly in an effort towards the achievement of hypersonic flight. At such high Mach numbers the stability or instability of the flow has many important consequences. For example it is desirable to reduce drag in external aerodynamical situations in order to increase performance, while it is imperative to increase instability in internal flows such as the scram jet-engine for example, so that mixing and thus propulsion are increased. Both the examples cited above contain certain fundamental aspects of wake flow instabilities and in this study a laminar two dimensional high-Reynolds-number wake is analyzed for stability with emphasis on the hypersonic limit.

The inviscid instability of a two-dimensional laminar compressible wake at high Reynolds number, was studied analytically and computationally by Papageorgiou (1989a). The general finding is that the flow stabilizes considerably as the Mach number is increased, with the largest spatial growth rates being attained for streamwise stations near the trailing edge. To give a numerical example, the maximum growth rate at a non-dimensional distance of 0.001 from the trailing edge, decreases from about 0.25 at zero Mach number to about 0.03 at Mach 3; at Mach 9 it decreases to 0.002. Similar drastic reductions in the instability are observed at all streamwise locations (see Papageorgiou (1989a)). It is clear, therefore, that the hypersonic limit merits a careful study for all streamwise locations. In particular the neutral stability curves give a wealth of information about the dominant physics, such as multiplicity of modes, mode crossing and dominant length-scales (for the related Blasius boundary-layer flow, see for example Mack (1987) and references therein, Cowley and Hall (1988), Smith and Brown (1989), Blackaby et al. (1990)).

Of great relevance to the present work, are the related studies of boundary- and shear-layer stability and in particular their hypersonic limit. Balsa and Goldstein (1989) consider hypersonic mixing layers, and in particular tanh velocity distributions, while Smith and Brown (1989) and more recently by Blackaby et al. (1990), consider the Blasius boundary-layer for a Chapman and Sutherland viscosity law fluid respectively. From all these studies, it emerges that the dominant mode of instability at hypersonic Mach numbers is the vorticity mode. This mode is localized around the generalized inflection point and decays away from it. Consequently, the majority of the boundary layer is passive in the description of this mode and the structure of the mode together with the neutral eigenvalues are

solely dependent on the asymptotic form of the basic flow far away from the wall (the inflection point is pushed to infinity as the Mach number becomes asymptotically large). Such a localized structure was also found by Hall and Fu (1989) in their study of hypersonic Görtler vortex instabilities. The asymptotic behavior of the basic flow at large distances from the wall becomes a function of the viscosity law used in the compressible boundary-layer equations. Smith and Brown use the Chapman viscosity law, while Blackaby et al. use the Sutherland model. In the former case the neutral wavespeed of the vorticity mode behaves like $\sqrt{\log(M^2)}$, while in the latter it goes to an order one constant at infinite Mach numbers. It is also worth noting that Smith and Brown found an exact solution to the reduced vorticity mode eigenvalue problem; the vorticity equation is a canonical equation, therefore, and applies to a range of flows. We show how it can be used to describe the hypersonic stability of a developing wake and also present the vorticity mode analysis for mixing layers and obtain the same result as in the analytical study of Balsa and Goldstein (1989); comparisons are also made with the numerical study of Jackson and Grosch (1989a).

In the present problem we work with the Chapman viscosity law. The case of the Sutherland law gives a more complicated base-flow and is still to be studied. A cartesian frame of coordinates is used which is fixed at the trailing edge $x = 0$. The flow quantities ρ , u , v , w , p , T , μ denote non-dimensional density, velocities in the x, y and z directions respectively, pressure, temperature and viscosity coefficient; non-dimensionalizations are made with respect to free-stream values denoted by a subscript ∞. The Reynolds number is defined as $R_e = \dfrac{U_\infty L}{v_\infty}$ where L is the plate length, the Mach number is $M = \dfrac{U_\infty}{a}$ where a is the speed of sound, and the Prandtl number is $P_r = \dfrac{\mu_\infty c_p}{k}$ where c_p is the specific heat at constant pressure and k is the coefficient of thermal diffusivity. The Reynolds number is taken to be asymptotically large throughout and the flow is assumed to be laminar. We also make the assumption of a model fluid with a linear viscosity law, i.e. $P_r = 1$ and $\mu = C\,T$ (Chapman's law), where C is a constant that is taken to be 1 the equation $\mu = T$ is used.

2. The governing equations.

If the Dorodnitsyn-Howarth transformation $Y = \int_0^y \dfrac{dv}{l}$ is employed (see Stewartson (1964)), the undisturbed flow which is solely in the x-direction to leading order (the transverse velocity is of order

$R_e^{-1/2}$), satisfies the following nonlinear system :

$$\overline{u} = \overline{\psi}_Y \quad , \quad \overline{u}\,\overline{u}_x - \overline{\psi}_x\,\overline{u}_Y = \overline{u}_{YYY} \quad , \tag{1}$$

$$\overline{T}(x,Y) = 1 + \frac{1}{2}(\gamma-1)\,M^2\,(1-\overline{u}^2) \quad . \tag{2}$$

Equations (1), (2) will be used in $x > 0$; (2) is an exact integral of the energy equation for unit Prandtl numbers under the Crocco assumption (Stewartson (1964)). The boundary conditions in the wake are,

$$\overline{u}(x,\infty) = 1 \quad , \quad \overline{\psi}(x,0) = \overline{u}_Y(x,0) = 0 \quad , \quad \overline{\psi}(0+\,,\,Y) = \psi_B(Y) \quad . \tag{3}$$

The function ψ'_B represents the Blasius boundary-layer velocity just upstream of the trailing edge, and is given by,

$$\psi_B''' + \frac{1}{2}\psi_B\,\psi_B'' = 0 \quad , \quad \psi_B(0) = \psi_B'(0) = \psi_B'(\infty) = 0 \; ; \; \psi_B''(0) = \lambda = 0.332... \quad .$$

The stability equation for the perturbation pressure eigenfunction $p(Y)$ is

$$p'' - \frac{2\overline{u}'}{\overline{u} - c}\,p' - \overline{T}^2\,[\,\alpha^2 + \beta^2 - \frac{\alpha^2}{\overline{T}}\,(\overline{u}-c)^2\,M^2\,]\,p = 0 \quad , \tag{4}$$

with boundary conditions

$$p'(0) = p(\infty) = 0 \quad or \quad p(0) = p(\infty) = 0 \quad . \tag{5}$$

We will work with the former boundary condition for comparison purposes with the boundary-layer work of Mack (1987), Cowley and Hall (1988), and Smith and Brown (1989). The physical difference between the different boundary conditions is that the former describes even disturbances while the latter describes odd ones (for quantitative differences see Papageorgiou (1989a)). Even though odd disturbances provide larger growth rates than the corresponding even ones for order one Mach numbers, this distinction is irrelevant in the hypersonic limit where the structure of the mode is obtained from a local analysis far away from the center-line (see later). It is also worth noting that the null condition at infinity is consistent with the physics of the problem since the sonic lines do not cross for any Mach number and so neutrally stable outgoing or incoming waves at infinity are not supported (see Papageorgiou (1989a) and Jackson and Grosch (1989b)).

3. Numerical solutions and results.

It can be seen that for a specified $\bar{u}(Y)$, (4) together with (5) is an eigenvalue problem that in general needs to be addressed numerically. In the neutral stability case, the value of the neutral wave-speed c_s say, is also known (see below) and the problem reduces to the determination of α and β which are now real and determine the wavelength and angle of propagation of the neutral wave. In what follows we take $\beta = 0$ and so are choosing to study two-dimensional disturbances.

Lees and Lin (1946), show that the neutral wavespeed is equal to the value of the undisturbed flow at generalized inflection points. These are given by the zeros, Y_c say, of the function $L(Y)$,

$$L(Y) = \frac{d}{dY}\left[\frac{\bar{u}_Y}{\bar{T}^2}\right] \quad . \tag{6}$$

In our problem there are two zeros symmetrically placed about the x-axis and, due to the symmetry of the problem, they give the same value of c_s. In solving (4) for $Y>0$, therefore, a singularity will be encountered at $Y = Y_c$. The integration contour is deformed around the singularity by extending $\bar{u}$ into the complex plane of Y. It is worth noting that for model flows such as the tanh shear-layer (Jackson and Grosch (1989a,b)) this analytic extension can be done *a priori* by simply replacing Y with an appropriate complex variable. Also in self-similar flows such as the Blasius boundary layer, the singularity in the stability equation can also be treated *a priori* by integration of the Blasius ordinary differential equation along a complex contour. In either case the eigenvalues of the problem are independent of the chosen contour, provided no branch cuts are encountered in the analytic extension. The present problem is more complicated, however, in that the base-flow is given by the solution of the nonlinear system (1); the numerical strategy is as follows.

First of all the undisturbed state is computed by numerical solution of (1) subject to (3). Since the equations are parabolic in x a marching procedure is used, with an adaptive grid near the trailing edge as dictated by the two-tiered singular structure of the flow there (a singularity appears in the pressure gradient and the normal velocity, necessitating the triple-deck viscous-inviscid interaction region (Stewartson (1969), Messiter (1970)). The numerical method together with results has been described elsewhere (Papageorgiou and Smith (1989), Papageorgiou (1989b)). For a particular streamwise station the position of the generalized inflection point Y_c is found from (6). Using a third order Taylor expansion about Y_c, the undisturbed velocity $\bar{u}(Y)$ is extended into the complex plane so that the contour

of integration goes around the singularity. A rectangular deformation was used which passes below the singularity in the complex plane. This is consistent with the zero viscosity limit solutions of the stability problem (Mack (1987)).

The eigenvalue problem was solved by a shooting method, starting at infinity (some large value of Y) where the asymptotic forms of the solution are used, and integrating inwards to $Y = 0$ using a fourth-order Runge-Kutta method, and by employing a root-finding routine to satisfy the boundary conditions. Accuracy and convergence tests were made so that the reported results are not dependent on either the deformed contour or the number of grid-points representing the basic flow.

The results of the calculations are shown in Figures 1-6. The difference between the figures is that they correspond to the neutral stability of the flow at successively increasing distances from the trailing edge. The distances are shown on the figures in terms of the variable ξ used in the base-flow computations and which is equal to $x^{1/3}$. Figures 1 and 2 corresponds to positions in the wake at non-dimensional distances of 0.000125 and 0.001 from the trailing edge. These are near-wake stations and it is not surprising to find that our results are almost the same as the equivalent ones for the Blasius boundary layer presented by Cowley and Hall (1988). The only structure that is different is the zero Mach number limit. This limit is inviscidly stable for boundary-layers but not for wakes (for a discussion of this limit in boundary-layers see Gajjar (1989), and for wake-flows see Papageorgiou and Smith (1989)). Once the flow becomes supersonic, however, the wake results shown here are a slight displacement of the boundary-layer ones. The explanation of this lies in the fact that the generalized inflection point for compressible supersonic boundary layers lies in the main part of the boundary-layer and so is outside the Goldstein layer (see Goldstein (1930)) which comprises a region of lateral extent of $O(x^{1/3})$ as $x \to 0$. The Goldstein layer is therefore passive and acts only to displace the inflection point by a lateral distance of $O(x^{1/3})$. At hypersonic Mach numbers agreement is even more pronounced since the inflection point (critical layer) now lies at a logarithmically large distance from $Y = 0$ (see next section also). The other feature of Figure 1 worth noting is the mode multiplicity at Mach numbers above approximately 3. This is in line with boundary-layer stability where these "second modes" were discovered by Mack (1965), (1984). As the Mach number increases, it is clear that a new and almost continuous mode emerges by different modes crossing. This is the wake equivalent to the vorticity mode analyzed by Smith and Brown (1989) and is characterized by neutral wavenumbers which grow like $\sqrt{\log(M^2)}$. The modes below the vorticity mode decay to zero with Mach number and

they are equivalent to the acoustic modes analyzed by Cowley and Hall (1988); they decay like M^{-2} at high Mach numbers.

Figure 3, corresponds to the wake-station $\xi = 0.4$ ($x = 0.064$). Even though we are still near the trailing edge, the basic flow is no longer sharply split into the two-tiered structure of Goldstein (1930), but the layer where the flow adjusts to symmetry about $Y = 0$ grows to order one values. The quantitative features of the neutral curves become much different from those of the Blasius boundary-layer. Multiple modes still occur but are now delayed to Mach numbers of approximately 4 and over. The vorticity mode is still clearly seen and so are the decaying acoustic modes. The maximum of the first mode increases and in general, for a given Mach number, the neutral wavenumber is increased as compared to the previous wake-station.

The fate of the multiple or higher modes, begins to become more evident in Figures 4 and 5 which correspond to $\xi = 0.6$, 0.8 ($x = 0.216$, 0.512) respectively. A second mode appears at a Mach number of approximately 5.5 and 7 respectively and at the same time the first and second mode come together. In Figure 4 the first and second modes almost cross at a Mach number of about 8.5. The presence of the vorticity and acoustic modes is still seen, even though their growth or decay respectively is slowed down somewhat and one needs to go to higher Mach numbers to see it. The vorticity mode (which is the dominant one at high Mach numbers) is described analytically in the next section.

Figure 6 shows a collection of the neutral curves computed for the wake-stations $\xi = 1.2$, 1.4, 1.6, 1.8 ($x = 1.728$, 2.744, 4.696, 5.832). For all these cases the Mach number range 0 to 20 was scanned for second modes but none were found. The second modes are present at much higher Mach numbers and are probably of little physical value then. It is worth noting that computations on the compressible tanh shear layer or the Lock profile (Jackson and Grosch (1989a,b)) do not reveal any second modes at moderately large Mach numbers at least. The reason for this is that both these mixing layer flows model the far-wake development of two compressible boundary-layers, with different slip velocities, coming together and mixing at the trailing edge. Undoubtedly, by analogy with the present problem, such modes exist in the developing region of the mixing layer before full mixing occurs. In fact the dominant neutral hypersonic mode that emerges for both symmetric wakes (the present problem) and the far-wake development of non-symmetric ones (tanh or Lock profiles), is the vorticity mode analogous to the one described by Balsa and Goldstein (1989) for mixing layers and, Smith and

Brown (1989) for boundary layers. This mode is analyzed in the next section for the wake and in an Appendix for the tanh and Lock shear layers.

4. The vorticity mode in the wake.

In this section we will show that the structure of the vorticity mode is essentially the same as that obtained for the Blasius boundary layer (see earlier references). The only complication is the fact that the undisturbed velocity in the streamwise direction does not possess a uniformly valid self-similar form, and so each streamwise station should be treated separately. As the hypersonic limit is approached, however, the critical layer gets pushed out to the edge of the wake boundary layer and so only the large Y behavior of the flow is required for any given position in the wake. As has been noted by previous investigators (Smith and Brown (1989), Cowley and Hall (1989), Blackaby et al. (1990)), the physical significance of the critical layer in the hypersonic limit, is that it provides a thin layer inside which the undisturbed temperature adjusts from an order M^2 value (note that $M \gg 1$) just below the layer, to its undisturbed $O(1)$ value just above it. The large temperatures near the plate are a consequence of heating due to the large Mach numbers involved, and for a boundary-layer on a semi-infinite flat plate for example, this increase in temperature takes place for all streamwise stations. The situation is different for a wake, however. When the no-slip condition at the trailing edge is removed, the undisturbed temperature has its maximum value at the wake center-line. A glance at the temperature equation (2), shows that as the far-wake limit is approached (the velocity $\bar{u}$ tends to its undisturbed value of 1), the base-flow temperature tends to its ambient unit value independent of the Mach number. The adjustment layer will no longer be thin but will occupy the whole wake. This limit is more subtle than the description at order one wake-stations and we describe it first. It will also be shown how the far-wake limit, which appears to have a different structure, produces the vorticity mode seen upstream. This is done by taking the zero limit of the local scaled far-wake streamwise coordinate as shown below.

For $x \gg 1$, then, the basic flow takes the form

$$\bar{u} = 1 - \frac{A}{x^{1/2}} e^{-Y^2/x} + \cdots \quad , \quad \bar{T} = 1 + \frac{A}{x^{1/2}} M^2 (\gamma-1) e^{-Y^2/x} + \cdots \quad , \tag{7}$$

where A is a positive constant (see Rosenhead (1963), Schlichting (1948)). These results are readily obtainable from (1), (2) by linearizing $\bar{u}$ and $\bar{T}$ about unity. As explained above, the vorticity mode is

obtained in regions where $\bar{T}$ is of order one. The following scalings are introduced, therefore.

$$x = M^2 \, \bar{x} \quad , \quad Y = M^2 \, \bar{Y} \quad , \quad Y^2 / x = \bar{Y}^2 / \bar{x} = \eta^2 \quad , \tag{8}$$

where tilde variables are now of order one. The basic flow becomes

$$\bar{u} = 1 - \frac{1}{M^2 \, \bar{x}^{1/2}} \, e^{-\bar{Y}^2/\bar{x}} \quad , \quad \bar{T} = 1 + \bar{x}^{-1/2} \, (\gamma - 1) \, e^{-\bar{Y}^2/\bar{x}} \quad . \tag{9}$$

The wavespeed c also expands as

$$c = 1 - \frac{c_1}{M^2} + \cdots \quad ,$$

a result that follows from the evaluation of $\bar{u}$ in the critical layer. Substitution into the Rayleigh equation (4) gives

$$\frac{d^2p}{d\bar{Y}^2} - \frac{4\bar{Y}e^{-\bar{Y}^2/\bar{x}}}{\bar{x}^{3/2}(c_1 - \bar{x}^{-1/2}e^{-\bar{Y}^2/\bar{x}})} \, \frac{dp}{d\bar{Y}} - \bar{\alpha}^2 \, (1 + \bar{x}^{-1/2}(\gamma - 1)e^{-\bar{Y}^2/\bar{x}}) \, p = 0 \quad . \tag{10}$$

The wavenumber α in (4) has been scaled according to

$$\alpha = \frac{\bar{\alpha}}{M^2} \quad ,$$

and $\bar{\alpha}$ is the order one quantity that appears in (10) above. Equation (4.4) is to be solved subject to decay as $\bar{Y} \to \pm\infty$.

The interesting limit that matches the solutions of (10) to those upstream is obtained by taking $\bar{x} \to 0$. As $\bar{x}$ becomes small, the value of $\bar{Y}$ also becomes small, and its functional dependence is obtained from the balance

$$\bar{x}^{-1/2}e^{-\bar{Y}^2/\bar{x}} = O(1) \quad . \tag{11}$$

This balance makes all the terms in (10) of the same order, which is essential in producing the vorticity mode upstream. If we solve (11) we find that to leading order $\bar{Y}$ goes to zero like $\sqrt{-(1/2)\bar{x}\,log(\bar{x})}$. The next order approximation leads to the thickness of the layer and a new local coordinate $\bar{Y}$ given by

$$\bar{y} = L + \frac{\bar{x}\,\bar{Y}}{L} \quad , \quad L = \sqrt{-(1/2)log(\bar{x}^{1/2})} \quad .$$

Substitution into (10) yields

$$\frac{d^2p}{d\bar{Y}^2} - \frac{4e^{-2\bar{Y}}}{c_1 - e^{-2\bar{Y}}} \, \frac{dp}{d\bar{Y}} - \frac{\bar{x}^2\,\bar{\alpha}^2}{(-\bar{x}log(\bar{x}^{1/2}))} \, (1 + (\gamma - 1)e^{-2\bar{Y}}) \, p = 0 \quad . \tag{12}$$

The boundary conditions are that the disturbance vanishes as $|\bar{Y}| \to \infty$. The following rescaling of $\bar{\alpha}$ puts (12) into the canonical vorticity layer equation (Smith and Brown (1989)),

$$\bar{\alpha} = \frac{(-\bar{x}\log\bar{x}^{1/2})^{1/2}}{\bar{x}} \alpha_V \quad .$$

The solutions of (12) have been given by Smith and Brown and we need only quote the results here. If the transformation $s = (\gamma-1)e^{-2\bar{Y}}$ is made, together with $p = s^{\alpha_V}P(s)$, the exact solution $P = e^{-s/4}$ is found when c_1 takes on its neutral value from the inflection point condition, when α_V is given by

$$\alpha_V = \frac{1}{4} \quad .$$

Smith and Brown show also how higher order corrections to the neutral wavespeed can be generated and the reader is referred to that paper for a complete description of the vorticity mode.

If we now write the unscaled wavenumber α in terms of α_V we find

$$\alpha = \frac{1}{M^2} \frac{\sqrt{-\bar{x}\log\bar{x}^{1/2}}}{\bar{x}} \alpha_V \quad . \tag{13}$$

Now the local scale $\bar{x}$ was defined in (8); it can be seen, therefore, that for x to be of order (in other words to match with the solutions upstream), the scaling $\bar{x}^{1/2} \sim M^{-2}$ is relevant. Equation (13) now gives

$$\alpha = \frac{1}{4} \sqrt{(\log M^2)} \quad , \tag{14}$$

which is the vorticity mode result of Smith and Brown.

To summarize, we considered the stability problem of the far-wake in the double limit $M \gg x \gg 1$. The other limit $x \gg M \gg 1$ is much easier and is discussed below for completeness even though the result has already been found by letting $\bar{x} \to 0$ above.

For large x, therefore, the basic flow takes the form (7) above, where the vertical coordinate is $\eta = Y/x^{1/2}$. The generalized inflection points are the zeros of

$$\bar{u}'' \bar{T} = 2 \bar{u}' \bar{T}' \quad ,$$

where primes denote η derivatives and x is effectively a parameter. The inflection point occurs as $\eta \to \infty$, and it is easy to show that its position is given by the solution of

$$\frac{M^2}{x^{1/2}} e^{-\frac{1}{4}\eta^2} = O(1) \quad .$$

Note that we assume that $M^2 \gg x^{1/2}$; the case when these are in balance has been discussed above. To study the vorticity layer we write

$$\frac{1}{2}\eta = \Gamma + \frac{\overline{\eta}}{2\Gamma} \quad , \quad \frac{M^2}{x^{1/2}} e^{-\Gamma^2} = 1 \quad ,$$

as in Smith and Brown (1989), Cowley and Hall (1988). The inflection point then occurs at $\eta_s = log(A(\gamma-1))$. Expanding c as before and making the transformation $Ae^{-\overline{\eta}} = e^{-z}$ together with the rescaling $\alpha = \Gamma\alpha_V$ gives the canonical vorticity equation described above. The exact neutral solution of Smith and Brown is as above and the final result is

$$\alpha = \frac{1}{4}\left[log\frac{M^2}{x^{1/2}}\right]^{1/2} \quad . \tag{15}$$

It is exactly this mode structure that was obtained in the previous case in the limit of small $\bar{x}$.

At order one distances from the trailing edge, the behavior of the basic flow at large Y has the quadratic exponential decay of (7) for example. This is enough to dictate that the vorticity mode structure has wavenumbers that scale with $\sqrt{logM^2}$.

To see the importance of the asymptotic behavior of the flow at infinity in determining the dominant hypersonic modes of instability, we include in an Appendix the hypersonic stability of two profiles that are widely used in mixing layer studies, the hyperbolic tangent and Lock velocity distributions. It is shown that completely different results arise for the two cases.

5. Conclusions.

We have carried out a computational and analytical study of the inviscid instabilities present in laminar wake flows. We have computed the variation of neutral wavenumbers with Mach number for several wake-stations. It has been found that near-wake supersonic modes are essentially a small perturbation of those found for the Blasius boundary-layer, the reason being that inviscid compressible neutral waves are associated with the generalized inflection points of the basic flow, and these points are almost coincident for the near-wake and the Blasius profile. As the wake develops, the Mach number range which supports a unique mode increases. Higher modes are delayed to ever increasing

Mach numbers and for fixed but large Mach numbers the rate of growth of the vorticity mode recedes and so does the rate of decay of the acoustic modes. This plays a role in the comparison of analytical results of the vorticity modes and the computed ones. The agreement is excellent at near-wake stations but as the wake develops very high Mach numbers are required to bring exact and asymptotic results into agreement. The asymptotic result is valid at infinite Mach numbers anyway and the neutral wavenumber α grows fairly slowly with Mach number due to the logarithmic dependence described in the previous section. Our results indicate that even though the vorticity mode clearly emerges as the major hypersonic instability mechanism, its asymptotic description for wake stations far from the trailing edge comes into play at very large Mach numbers that are very dependent on the station under consideration. This may not be the case for other flows, and indeed as we show in the Appendix, the vorticity mode analysis fully captures the developed mixing layer dynamics.

Our results also indicate that multiplicity of modes is delayed to impractically large Mach numbers for positions far enough from the trailing edge. At the wake station which is about one plate-length away from the trailing edge, there is only one mode present for the range $0 \leq M \leq 20$. This is certainly also true for distances further downstream. We note here that it is difficult to continue the computations to Mach numbers beyond approximately 20, because the neutral wavespeed tends to the free stream velocity making the solutions inaccurate.

Finally, it is interesting to consider the effects of the thermodynamics used in the model, and in particular the relaxation of the Chapman linear viscosity law. A more accurate relationship is believed to be the Sutherland law $\mu = \left[\dfrac{1+C}{\overline{T}+C}\right]\overline{T}^{3/2}$ where C is a constant approximately equal to $110^0 K / T_\infty$. This model was adopted by Blackaby et al. (1990) in their interesting study of hypersonic flow past a flat plate. They find that the basic flow decays algebraically at infinity and as a consequence the structure of the vorticity mode is significantly altered in that the neutral wavespeed approaches a constant in the hypersonic limit, rather than exhibit the Mach number dependent growth obtained from the Chapman law. It should be noted, however, that the maximum growth rate is still small and of order M^{-2}. We expect similar results to hold for a developing wake with the Sutherland viscosity law. Some preliminary work indicates that the far-wake has the vorticity mode structure of the Chapman law Blasius flow, while at near-wake stations the results of Blackaby et al. are expected. The interesting regime of order one streamwise stations must be tackled numerically, it seems, since there is no simple

representation of the basic flow.

Appendix.

In this section we briefly present the analysis that leads to the hypersonic stability limit of two model mixing layers. For the tanh mixing layer, we will arrive at the result given by Balsa and Goldstein (1989), but in a more straight-forward manner. This result is also consistent with the computations of Jackson and Grosch (1989a).

The tanh mixing layer.

We take the basic flow used by Jackson and Grosch (1989) and Balsa and Goldstein (1989), the stability equation being (4).

$$\bar{u}(\eta) = \frac{1}{2}(1 + \tanh\eta) \quad , \tag{A1}$$

$$\bar{T}(\eta) = 1 - (1-\beta_T)(1-\bar{u}) + \frac{1}{2}(\gamma-1)M^2\bar{u}(1-\bar{u}) \quad . \tag{A2}$$

Using (A1) and (A2) together with (6) we find that the inflection point position satisfies the equation

$$\frac{1}{8}(\gamma-1)M^2 - \cosh^2\eta = \frac{1}{2}(1-\beta_T)\frac{\cosh^2\eta}{\sinh\eta}(\cosh\eta-\sinh\eta) \quad . \tag{A3}$$

It is clear from (A3) that as M becomes infinite the value of η must increase like $\log M$. More precisely, solving (A3) for large η we find

$$\eta = \log M + \frac{1}{2}\log\left[\frac{\gamma-1}{2}\right] + O(\frac{1}{M^2}) \quad . \tag{A4}$$

To study the behavior of the critical layer, therefore, we introduce the $O(1)$ variable $\bar{\eta}$ given by

$$\eta = \log M + \bar{\eta} \quad .$$

The wavespeed is expanded also

$$c = 1 - \frac{c_1}{M^2} + \dots,$$

where the neutral value is given by $c_1 = 2/(\gamma-1)$. Substitution into (A1), (A2) and then into the Rayleigh equation (4), together with the transformation $e^{-2\bar{\eta}} = 2e^{-z}$ gives

$$\frac{d^2p}{dz^2} - \frac{2e^{-z}}{\frac{c_1}{2} - e^{-z}}\frac{dp}{dz} - \frac{1}{4}\alpha^2(1 + (1-\gamma)e^{-z})^2 p = 0 \quad . \tag{A5}$$

The Smith and Brown transformation described in section 3 applies, and so does the exact neutral solution they found. The result is

$$\alpha = \frac{1}{2} \quad , \tag{A6}$$

in complete agreement with Balsa and Goldstein (1989).

The Lock profile.

The tanh velocity distribution discussed above is only a model for the mixing layer and does not satisfy the equations of motion. It is meant to model the solution derived by Lock (1951) in his study of the boundary layer between parallel streams. The distribution is found as a similarity solution of the boundary-layer equations with a similarity variable $\eta = Y/2x^{1/2}$.

$$\bar{u}(\eta) = f'(\eta) \quad , \quad f''' + 2ff'' = 0 \quad , \quad \bar{u}(\infty) = 1 \quad , \quad \bar{u}(-\infty) = 0 \quad , \tag{A7}$$

$$\bar{T} = 1 - (1-\beta_T)(1-\bar{u}) + \frac{1}{2}(\gamma-1)M^2\bar{u}(1-\bar{u}) \quad , \tag{A8}$$

where β_T is the ambient temperature of the lower stream. Using (A7), (A8) we find that the generalized inflection point position satisfies the equation

$$f''' - (1-\beta_T)\left[f''(1-f) - 2f'^2 \right]$$

$$= (\gamma-1)M^2\left[f'^2(1-2f) - \frac{1}{2}f''(1-f)f' \right] \quad . \tag{A9}$$

The behavior of $\bar{u}$ as $\eta \to \infty$ is therefore the same as that for the Blasius boundary-layer (e.g. Rosenhead (1963)). The asymptotic form is given by

$$\bar{u} \sim 1 - \lambda_1 \left(\zeta^{-1} - \zeta^{-3} + O(\zeta^{-5}) \right) exp(-\frac{1}{2}\zeta^2) \quad , \quad \zeta = \eta - \lambda_2 \quad ,$$

where λ_1 and λ_2 are positive constants we need not specify here. For large η therefore, the dominant balance in (A9) comes from the third derivative terms and gives the critical layer scaling

$$\zeta = \Gamma + \frac{\zeta}{\Gamma} \quad ,$$

with

$$(\gamma-1)\lambda_1 \frac{e^{-\Gamma^2}}{\Gamma} = \frac{1}{M^2} \quad .$$

To leading order, therefore, $\Gamma = \sqrt{(\log M^2)}$, in line with the result of Smith and Brown as expected. The neutral wavenumber is found by deriving the vorticity equation as above, and utilizing the exact solution of Smith and Brown. The final result is

$$\alpha = \frac{1}{2}\sqrt{(\log M^2)} \quad . \tag{A10}$$

It is interesting to compare the theoretical results (A6) and (A10) with the computations of Jackson and Grosch (1989a). For example, their Figure 13, gives the neutral curves of α versus Mach number for the tanh and Lock profile as well as those for a Sutherland viscosity law, for the subsonic fast modes which have been analyzed here. It is clear from the figure that the neutral curve for the tanh profile is tending to a constant. The value attained at the highest computed Mach number of 7 is about 0.35-0.4, and the curve is flat and increasing slightly. The same is true for the Sutherland viscosity law. The situation for the Lock profile is quite different, however. The neutral value of α at $M = 7$ is about 0.7 and the slope of the curve is not flat and is increasing slowly with Mach number. The leading order asymptotic result (A10) evaluated at $M = 7$ gives a value of approximately 0.986. The agreement becomes better when the Mach number is increased; further more, the next order correction to (A10) which is of order $\dfrac{\log(\log M^2)}{\log M^2}$ (see Smith and Brown) is negative and enhances agreement between the exact and asymptotic results.

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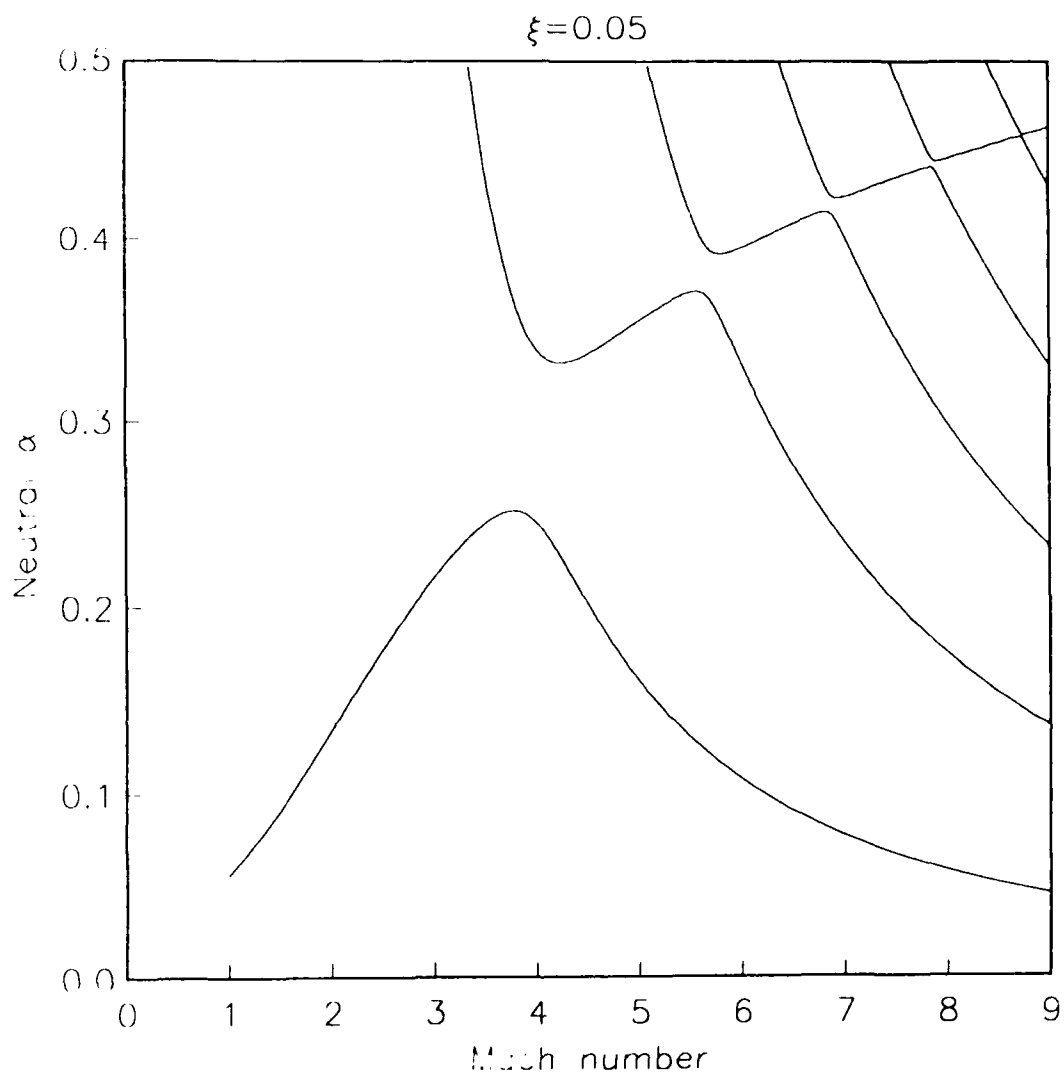
Stewartson, K. 1969 *Mathematika*, 16, pp.106-121.

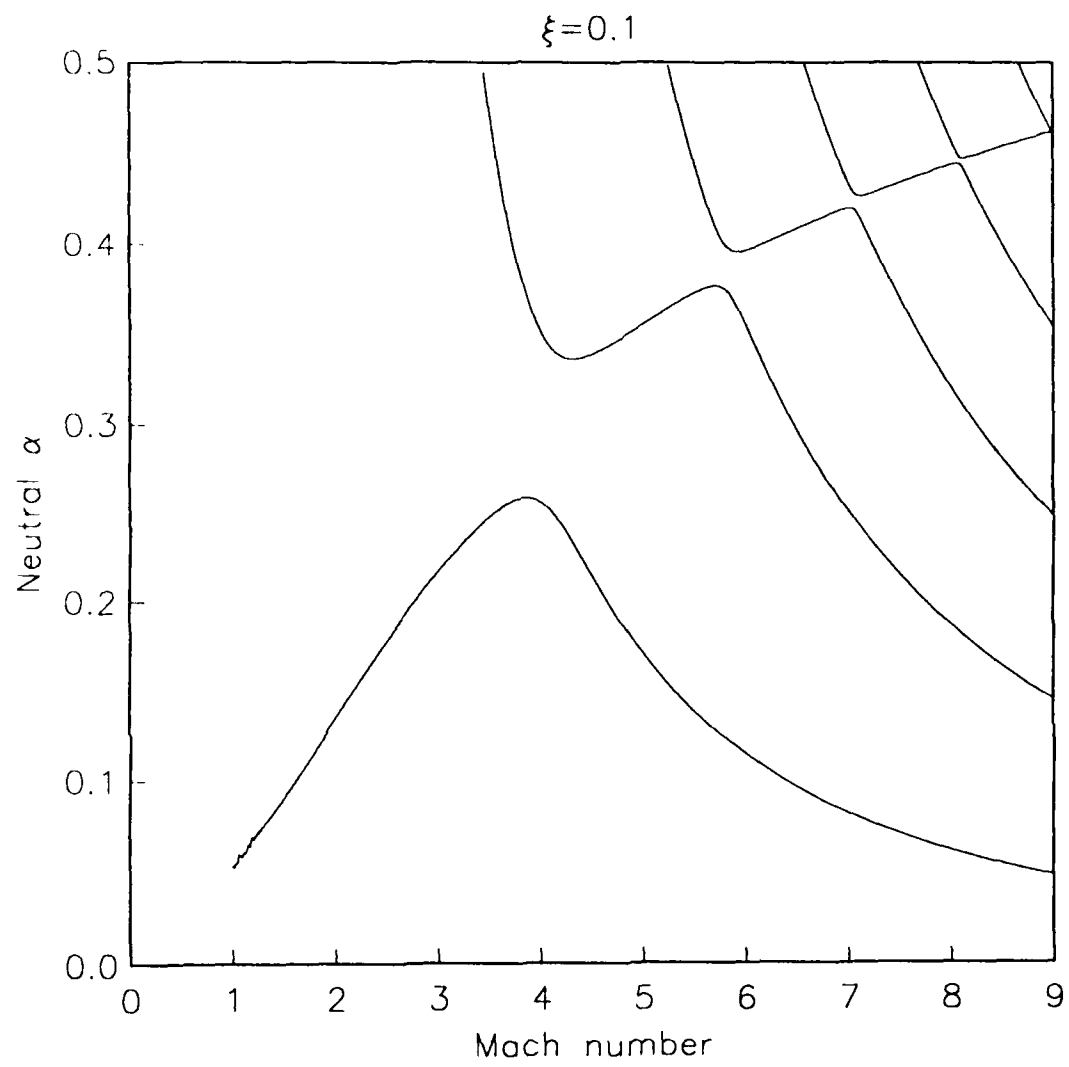FIGURE 1: Neutral stability curves at $\xi = 0.05$ (x=0.000125).
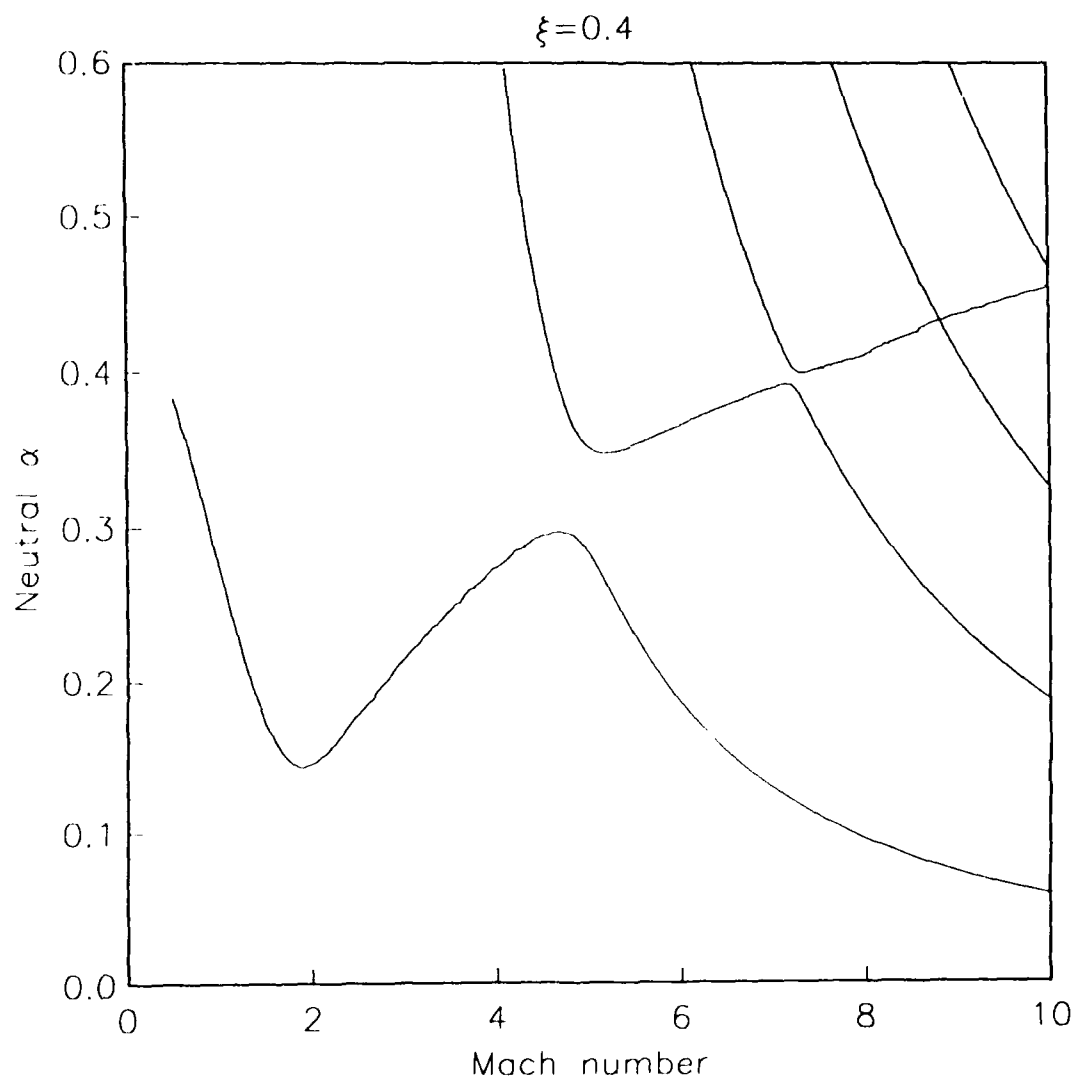
FIGURE 2: Neutral stability curves at $\xi = 0.1$ (x=0.001).

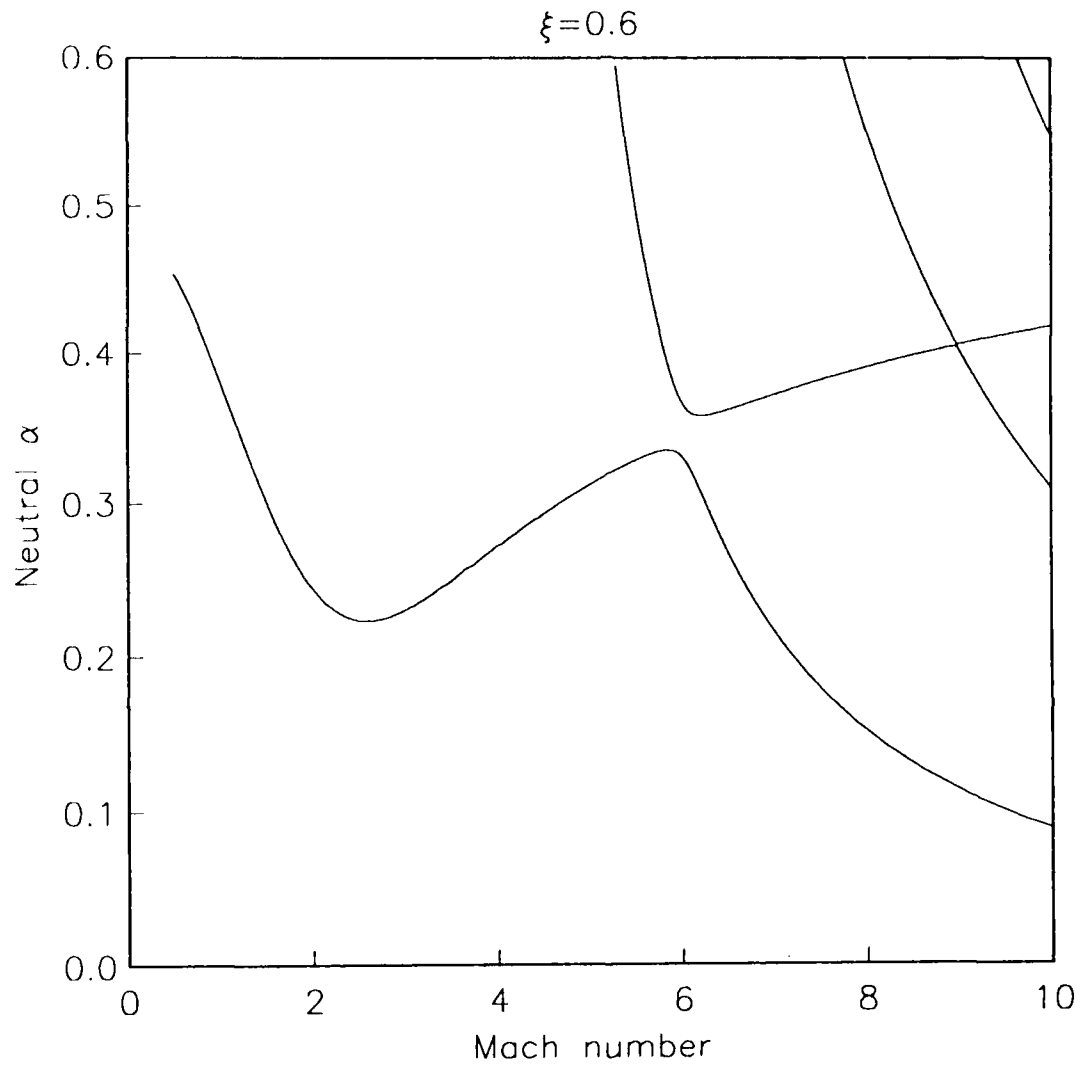FIGURE 3: Neutral stability curves at $\xi = 0.4$ (x≈0.064).

FIGURE 4: Neutral stability curves at $\xi = 0.6$ (x–0.216).
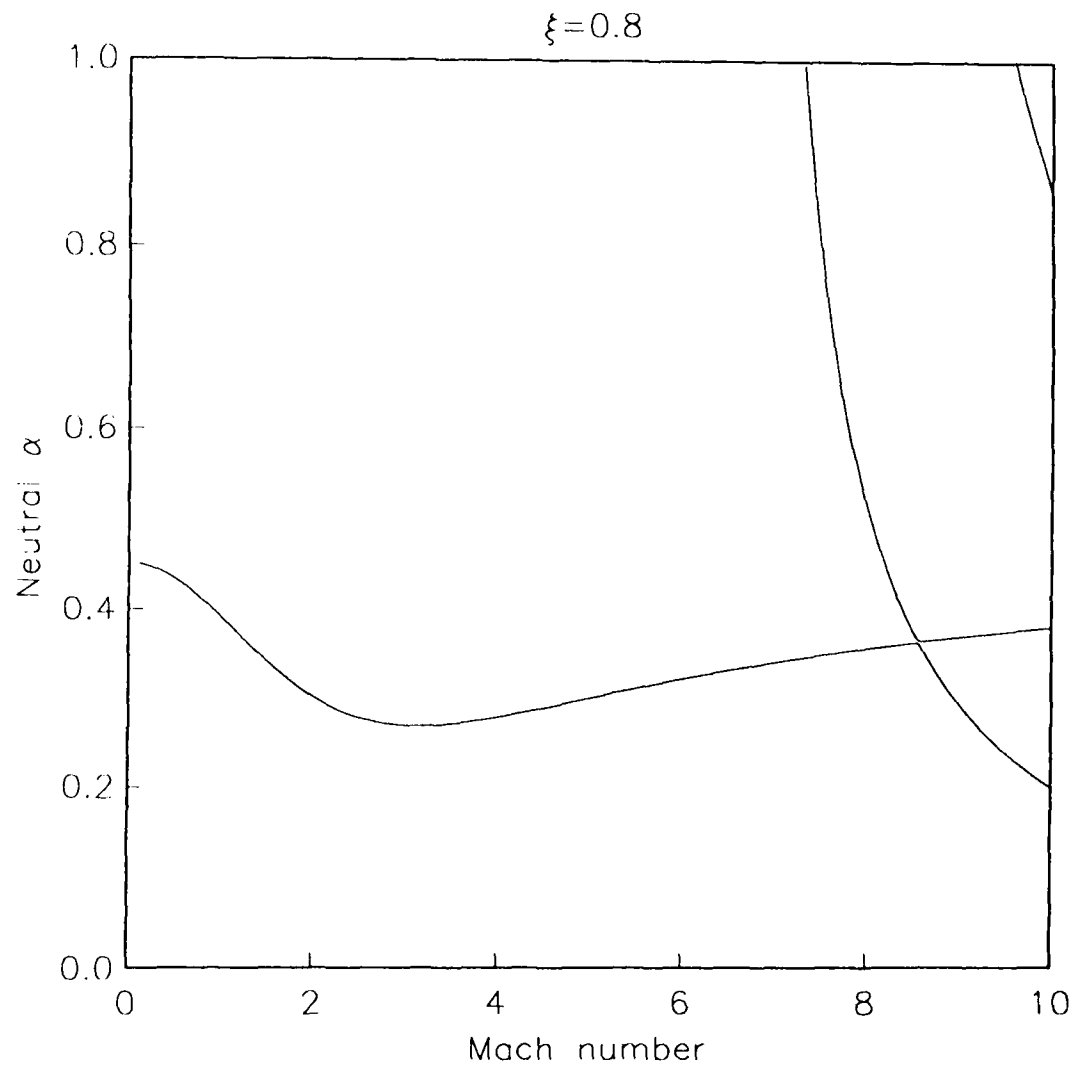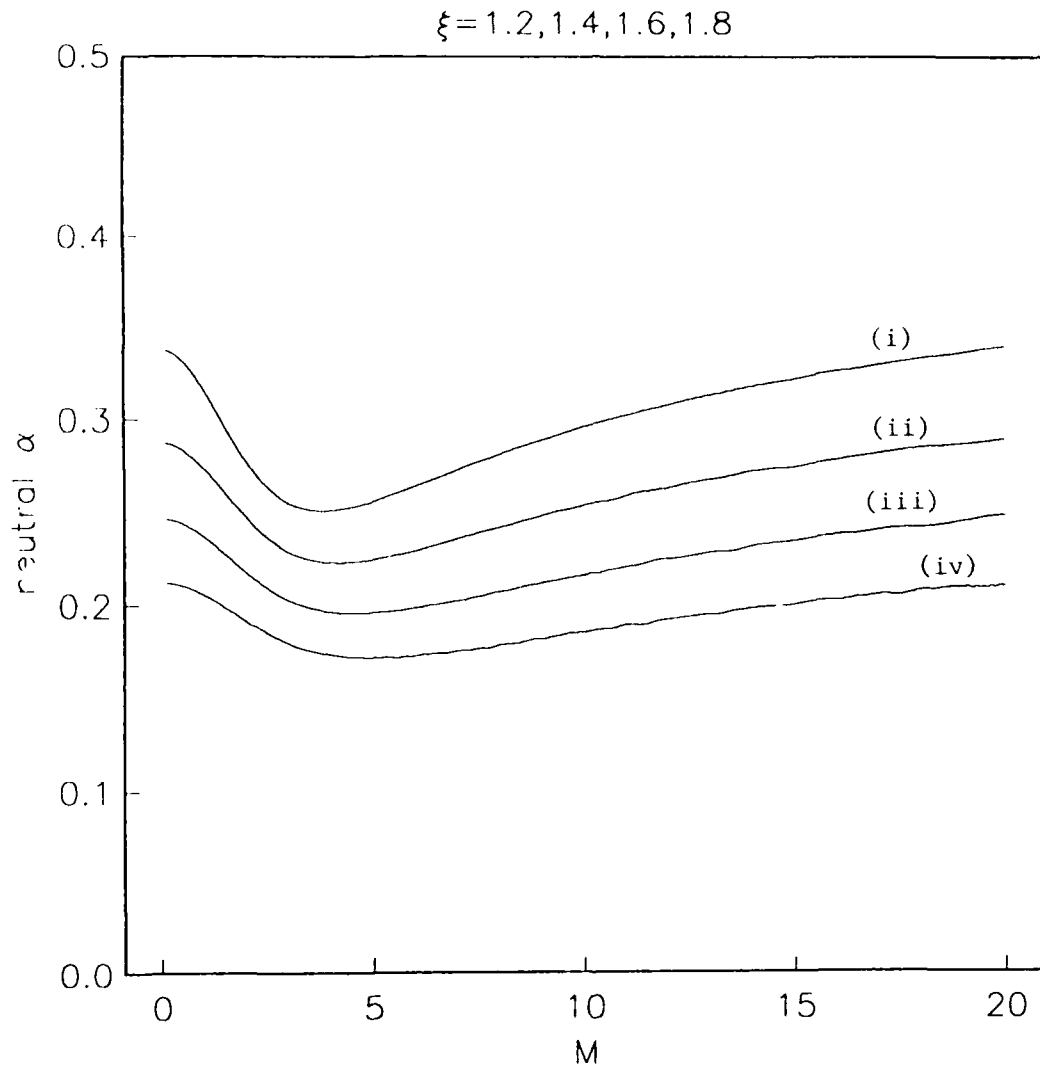
FIGURE 5: Neutral stability curves at $\xi = 0.8$ (x=0.512).

FIGURE 6: Neutral stability curves. (i) $\xi = 1.2$ (x=1.728), (ii) $\xi = 1.4$ (x=2.744), (iii) $\xi = 1.6$ (x=4.696), (iv) $\xi = 1.8$ (x=5.832).

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16. Abstract

This study is concerned with the stability properties of laminar free shear-layer flows, and in particular symmetric two-dimensional wakes, for the subsonic through the hypersonic regime. Emphasis is given to the use of proper wake profiles that satisfy the equations of motion at high Reynolds numbers. In particular we study the inviscid stability of a developing two-dimensional wake as it decelerates at the trailing edge of a splitter plate. The non-parallelism of the flow is a leading order effect, and the undisturbed state is solved numerically. The neutral stability characteristics are computed numerically and the hypersonic stability is obtained by increasing the Mach number. It is found that the neutral stability characteristics are altered significantly as the wake develops. Multiple modes (second modes) are found in the near-wake (they are shown to be closely related to the corresponding Blasius ones), but as the wake develops mode multiplicity is delayed to higher and higher Mach numbers. At a distance of about one plate length from the trailing edge, there is only one mode in a Mach number range of zero to twenty. The dominant mode emerging at all wake stations and for high enough Mach numbers is the so-called vorticity mode, which is centered around the generalized inflection point layer. The structure of the dominant mode is also obtained analytically for all streamwise wake locations and it is shown how the far-wake limit is approached. Asymptotic results for the hypersonic mixing layer given by a tanh and a Lock distribution are also given.

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